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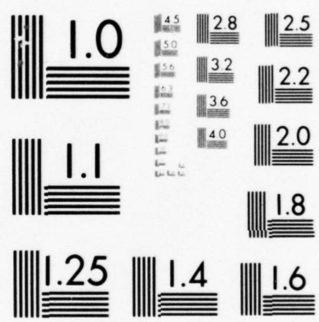

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THE DIFFICULT CHOICE

BY

A. J. DURELLI

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CONTRACT NO. N00014-76-C-0487
O.U. PROJECT NO. 30339-49
REPORT NO. 44

SCHOOL OF ENGINEERING
OAKLAND UNIVERSITY
ROCHESTER, MICHIGAN 48063

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ABSTRACT

It is necessary to weigh the advantages and limitations of the numerous methods available today for the analyses of displacements, strain, and stresses. Systematic studies are lacking on this subject. In this paper after comments are made on several theoretical approaches, in particular approximates methods, attention is concentrated on experimental methods and considerations are made on the relative merits of: 1) photoelasticity, moiré, brittle and photoelastic coatings, gages, grids, holography and speckle to solve two-dimensional problems; 2) photoplasticity and moiré to solve problems in plasticity; 3) photo-ortho-elasticity, photoelastic coatings and moiré to solve anisotropic problems; 4) the methods used to solve problems in dynamic photoelasticity; 5) the methods used in three-dimensional photoelasticity, and 6) projected gratings, shadow-moiré and holography to analyze bent plates. General considerations are also made on the relative merit of whole-field methods, in comparison with point-by-point methods. Finally, an attempt is made at evaluating merit of the finite element method in respect to the experimental stress analysis methods.

Advantages of particular methods and instruments are emphasized in the paper: whole-field methods, diffused light polariscopes, moiré to solve

problems in plasticity and in anisotropy, repeated flashes to solve reproducible problems in dynamic photoelasticity. It is also shown that in general the "freezing" method is the most practical and the most precise to solve three-dimensional problems. The range of usefulness of projected gratings, shadow moiré and holography to analyze plates is indicated. It is pointed out that the finite element method has replaced, or will soon replace, two-dimensional static photoelasticity and will probably also do so in fracture dynamics applications. On the other hand, photoelasticity competes frequently with advantage with the finite element method, in the solution of wave propagation problems, optimization problems, and three-dimensional problems. The potential of the method of "speckle" is pointed out.

Previous Technical Reports to the Office of Naval Research

1. A. J. Durelli, "Development of Experimental Stress Analysis Methods to Determine Stresses and Strains in Solid Propellant Grains"--June 1962. Developments in the manufacturing of grain-propellant models are reported. Two methods are given: a) cementing routed layers and b) casting.

2. A. J. Durelli and V. J. Parks, "New Method to Determine Restrained Shrinkage Stresses in Propellant Grain Models"--October 1962. The birefringence exhibited in the curing process of a partially restrained polyurethane rubber is used to determine the stress associated with restrained shrinkage in models of solid propellant grains partially bonded to the case.

3. A. J. Durelli, "Recent Advances in the Application of Photoelasticity in the Missile Industry"--October 1962. Two- and three-dimensional photoelastic analysis of grains loaded by pressure and by temperature are presented. Some applications to the optimization of fillet contours and to the redesign of case joints are also included.

4. A. J. Durelli and V. J. Parks, "Experimental Solution of Some Mixed Boundary Value Problems"--April 1964. Means of applying known displacements and known stresses to the boundaries of models used in experimental stress analysis are given. The application of some of these methods to the analysis of stresses in the field of solid propellant grains is illustrated. The presence of the "pinching effect" is discussed.

5. A. J. Durelli, "Brief Review of the State of the Art and Expected Advance in Experimental Stress and Strain Analysis of Solid Propellant Grains"--April 1964. A brief review is made of the state of the experimental stress and strain analysis of solid propellant grains. A discussion of the prospects for the next fifteen years is added.

6. A. J. Durelli, "Experimental Strain and Stress Analysis of Solid Propellant Rocket Motors"--March 1965. A review is made of the experimental methods used to strain-analyze solid propellant rocket motor shells and grains when subjected to different loading conditions. Methods directed at the determination of strains in actual rockets are included.

7. L. Ferrer, V. J. Parks and A. J. Durelli, "An Experimental Method to Analyze Gravitational Stresses in Two-Dimensional Problems"--October 1965. Photoelasticity and moiré methods are used to solve two-dimensional problems in which gravity-stresses are present.

8. A. J. Durelli, V. J. Parks and C. J. del Rio, "Stresses in a Square Slab Bonded on One Face to a Rigid Plate and Shrunk"--November 1965.
A square epoxy slab was bonded to a rigid plate on one of its faces in the process of curing. In the same process the photoelastic effects associated with a state of restrained shrinkage were "frozen-in." Three-dimensional photoelasticity was used in the analysis.

9. A. J. Durelli, V. J. Parks and C. J. del Rio, "Experimental Determination of Stresses and Displacements in Thick-Wall Cylinders of Complicated Shape"--April 1966.
Photoelasticity and moiré are used to analyze a three-dimensional rocket shape with a star shaped core subjected to internal pressure.

10. V. J. Parks, A. J. Durelli and L. Ferrer, "Gravitational Stresses Determined Using Immersion Techniques"--July 1966.
The methods presented in Technical Report No. 7 above are extended to three-dimensions. Immersion is used to increase response.

11. A. J. Durelli and V. J. Parks. "Experimental Stress Analysis of Loaded Boundaries in Two-Dimensional Second Boundary Value Problems"--February 1967.
The pinching effect that occurs in two-dimensional bonding problems, noted in Reports 2 and 4 above, is analyzed in some detail.

12. A. J. Durelli, V. J. Parks, H. C. Feng and F. Chiang, "Strains and Stresses in Matrices with Inserts,"-- May 1967.
Stresses and strains along the interfaces, and near the fiber ends, for different fiber end configurations, are studied in detail.

13. A. J. Durelli, V. J. Parks and S. Uribe, "Optimization of a Slot End Configuration in a Finite Plate Subjected to Uniformly Distributed Load,"--June 1967.
Two-dimensional photoelasticity was used to study various elliptical ends to a slot, and determine which would give the lowest stress concentration for a load normal to the slot length.

14. A. J. Durelli, V. J. Parks and Han-Chow Lee, "Stresses in a Split Cylinder Bonded to a Case and Subjected to Restrained Shrinkage,"--January 1968.
A three-dimensional photoelastic study that describes a method and shows results for the stresses on the free boundaries and at the bonded interface of a solid propellant rocket.

15. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--August 1968.
This report has been written following a trip conducted by the author through several European countries. A list is given of many of the laboratories doing important experimental stress analysis work and of the people interested in this kind of work. An attempt has been made to abstract the main characteristics of the methods used in some of the countries visited.

16. V. J. Parks, A. J. Durelli and L. Ferrer, "Constant Acceleration Stresses in a Composite Body"--October 1968.
Use of the immersion analogy to determine gravitational stresses in two-dimensional bodies made of materials with different properties.

17. A. J. Durelli, J. A. Clark and A. Kochev, "Experimental Analysis of High Frequency Stress Waves in a Ring"--October 1968.
A method for the complete experimental determination of dynamic stress distributions in a ring is demonstrated. Photoelastic data is supplemented by measurements with a capacitance gage used as a dynamic lateral extensometer.

18. J. A. Clark and A. J. Durelli, "A Modified Method of Holographic Interferometry for Static and Dynamic Photoelasticity"--April 1968.
A simplified absolute retardation approach to photoelastic analysis is described. Dynamic isopachics are presented.

19. J. A. Clark and A. J. Durelli, "Photoelastic Analysis of Flexural Waves in a Bar"--May 1969.
A complete direct, full-field optical determination of dynamic stress distribution is illustrated. The method is applied to the study of flexural waves propagating in a urethane rubber bar. Results are compared with approximate theories of flexural waves.

20. J. A. Clark and A. J. Durelli, "Optical Analysis of Vibrations in Continuous Media"--June 1969.
Optical methods of vibration analysis are described which are independent of assumptions associated with theories of wave propagation. Methods are illustrated with studies of transverse waves in prestressed bars, snap loading of bars and motion of a fluid surrounding a vibrating bar.

21. V. J. Parks, A. J. Durelli, K. Chandrashekhara and T. L. Chen, "Stress Distribution Around a Circular Bar, with Flat and Spherical Ends, Embedded in a Matrix in a Triaxial Stress Field"--July 1969.
A Three-dimensional photoelastic method to determine stresses in composite materials is applied to this basic shape. The analyses of models with different loads are combined to obtain stresses for the triaxial cases.

22. A. J. Durelli, V. J. Parks and L. Ferrer, "Stresses in Solid and Hollow Spheres Subjected to Gravity or to Normal Surface Tractions"--October 1969.
The method described in Report No. 10 above is applied to two specific problems. An approach is suggested to extend the solutions to a class of surface traction problems.

23. J. A. Clark and A. J. Durelli, "Separation of Additive and Subtractive Moiré Patterns"--December 1969.
A spatial filtering technique for adding and subtracting images of several gratings is described and employed to determine the whole field of Cartesian shears and rigid rotations.

24. R. J. Sanford and A. J. Durelli, "Interpretation of Fringes in Stress-Holo-Interferometry"--July 1970.
Errors associated with interpreting stress-holo-interferometry patterns as the superposition of isopachics (with half order fringe shifts) and isochromatics are analyzed theoretically and illustrated with computer generated holographic interference patterns.

25. J. A. Clark, A. J. Durelli and P. A. Laura, "On the Effect of Initial Stress on the Propagation of Flexural Waves in Elastic Rectangular Bars"--December 1970.
Experimental analysis of the propagation of flexural waves in prismatic, elastic bars with and without prestressing. The effects of prestressing by axial tension, axial compression and pure bending are illustrated.

26. A. J. Durelli and J. A. Clark, "Experimental Analysis of Stresses in a Buoy-Cable System Using a Birefringent Fluid"--February 1971.
An extension of the method of photoviscous analysis is presented which permits quantitative studies of strains associated with steady state vibrations of immersed structures. The method is applied in an investigation of one form of behavior of buoy-cable systems loaded by the action of surface waves.

27. A. J. Durelli and T. L. Chen, "Displacements and Finite-Strain Fields in a Sphere Subjected to Large Deformations"--February 1972.
Displacements and strains (ranging from 0.001 to 0.50) are determined in a polyurethane sphere subjected to several levels of diametral compression. A 500 lines-per-inch grating was embedded in a meridian plane of the sphere and moiré effect produced with a non-deformed master. The maximum applied vertical displacement reduced the diameter of the sphere by 27 per cent.

28. A. J. Durelli and S. Machida, "Stresses and Strain in a Disk with Variable Modulus of Elasticity"--March 1972.
A transparent material with variable modulus of elasticity has been manufactured that exhibits good photoelastic properties and can also be strain analyzed by moiré. The results obtained suggests that the stress distribution in the homogeneous disk. It also indicates that the strain fields in both cases are very different, but that it is possible, approximately, to obtain the stress field from the strain field using the value of E at every point, and Hooke's law.

29. A. J. Durelli and J. Buitrago, "State of Stress and Strain in A Rectangular Belt Pulled Over a Cylindrical Pulley"--June 1972.
Two- and three-dimensional photoelasticity as well as electrical strain gages, dial gages and micrometers are used to determine the stress distribution in a belt-pulley system. Contact and tangential stress for various contact angles and friction coefficients are given.

30. T. L. Chen and A. J. Durelli, "Stress Field in a Sphere Subjected to Large Deformations"--June 1972.
Strain fields obtained in a sphere subjected to large diametral compressions from a previous paper were converted into stress fields using two approaches. First, the concept of strain-energy function for an isotropic elastic body was used. Then the stress field was determined with the Hookean type natural stress-natural strain relation. The results so obtained were also compared.

31. A. J. Durelli, V. J. Parks and H. M. Hasseem, "Helices Under Load"--July 1973.
Previous solutions for the case of close coiled helical springs and for helices made of thin bars are extended. The complete solution is presented in graphs for the use of designers. The theoretical development is correlated with experiments.

32. T. L. Chen and A. J. Durelli, "Displacements and Finite Strain Fields in a Hollow Sphere Subjected to Large Elastic Deformations"--September 1973.
The same methods described in No. 27, were applied to a hollow sphere with an inner diameter one half the outer diameter. The hollow sphere was loaded up to a strain of 30 per cent on the meridian plane and a reduction of the diameter by 20 per cent.

33. A. J. Durelli, H. H. Hasseem and V. J. Parks, "New Experimental Method in Three-Dimensional Elastostatics"--December 1973.
A new material is reported which is unique among three-dimensional stress-freezing materials, in that, in its heated (or rubbery) state it has a Poisson's ratio which is appreciably lower than 0.5. For a loaded model, made of this material, the unique property allows the direct determination of stresses from strain measurements taken at interior points in the model.

34. J. Wolak and V. J. Parks, "Evaluation of Large Strains in Industrial Applications"--April 1974.
It was shown that Mohr's circle permits the transformation of strain from one axis of reference to another, irrespective of the magnitude of the strain, and leads to the evaluation of the principal strain components from the measurement of direct strain in three directions.

35. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--April 1975.
Continuation of Report No. 15 after a visit to Belgium, Holland, Germany, France, Turkey, England and Scotland.

36. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Linear and Non-linear Elastic and Plastic Strains in a Plate with a Big Hole Loaded Axially in its Plane"--July 1975.
Strain analysis of the ligament of a plate with a big hole indicates that both geometric and material non-linearity may take place. The strain concentration factor was found to vary from 1 to 2 depending on the level of deformation.

37. A. J. Durelli, V. Pavlin, J. O. Bühler-Vidal and G. Ome, "Elastostatics of a Cubic Box Subjected to Concentrated Loads"--August 1975.
Analysis of experimental strain, stress and deflection of a cubic box subjected to concentrated loads applied at the center of two opposite faces. The ratio between the inside span and the wall thickness was varied between approximately 5 and 121.

38. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Elastostatics of Cubic Boxes Subjected to Pressure"--March 1976.
Experimental analysis of strain, stress and deflections in a cubic box subjected to either internal or external pressure. Inside span-to-wall thickness ratio varied from 5 to 14.

39. Y. Y. Hung, J. D. Hovanesian and A. J. Durelli, "New Optical Method to Determine Vibration-Induced Strains with Variable Sensitivity After Recording"--November 1976.
A steady state vibrating object is illuminated with coherent light and its image slightly misfocused. The resulting specklegram is "time-integrated" as when Fourier filtered gives derivatives of the vibrational amplitude.

40. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Cyclic Stress Studies by Time-Averaged Photoelasticity"--November 1976.
"Time-averaged isochromatics" are formed when the photographic film is exposed for more than one period. Fringes represent amplitudes of the oscillating stress according to the zeroth order Bessel function.

41. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Time-Averaged Shadow Moiré Method for Studying Vibrations"--November 1976.
Time-averaged shadow moiré permits the determination of the amplitude distribution of the deflection of a steady vibrating plate.

42. J. Buitrago and A. J. Durelli, "On the Interpretation of Shadow-Moiré Fringes"--April 1977.
Possible rotations and translations of the grating are considered in a general expression to interpret shadow-moiré fringes and on the sensitivity of the method. Application to an inverted perforted tube.

43. J. der Hovanesian, "18th Polish Solid Mechanics Conference." Published in European Scientific Notes of the Office of Naval Research, in London, England, Dec. 31, 1976.
Comments on the planning and organization of, and scientific content of paper presented at the 18th Polish Solid Mechanics Conference held in Wisla-Jawornik from September 7-14, 1976.

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ACKNOWLEDGMENTS

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INTRODUCTION

The relative efficiency of the different methods that can be used to solve a stress analysis problem is seldom dealt with in books and articles in which new methods are developed or in which solutions to problems are obtained. Some years ago, the solution of boundary value problems in continuum mechanics was approached theoretically almost without exception by using "exact," closed form functions, and the only whole-field experimental method available was static two-dimensional photoelasticity. The question of selecting the best method among several possible alternatives did not come up.

Today, there are many theoretical methods available which are frequently used, most of them involving approximate procedures, and among them the so-called "finite element" method is of almost universal application. A systematic comparison, prepared before the development of the finite-element method, of the advantages and limitations of several numerical methods (collocation, Galerkin, least squares, etc.) used to obtain approximate solutions to specific stress analysis problems has been made by Crandall.(1) Shortly after Sternberg(2) published a review of solutions, obtained theoretically, of three-dimensional stress concentrations and later Neuber(3) in an extensive paper gave a long list of references to solutions of plane

stress concentrations problems. In both of these last two cases the emphasis was put on the survey of the literature rather than on the critical evaluation of the methodology. More recently a comprehensive review of the particular features of "functional" methods, and of numerical and finite-element methods was published by Rao[4] who had particularly in mind the application of these methods to the determination of stress concentrations in structures. The limitations of transform and integral equation techniques to solve problems in which boundaries are close to geometric discontinuities are pointed out. Also pointed out is the important limitation of approximate mapping functions when applied to stress determinations at sharp corner, since these functions have the tendency to distort the shape of the sharp corner into the shape of a smooth fillet.

The number of experimental methods has also grown over the years. There are several alternatives in dynamic photoelasticity and even in three-dimensional photoelasticity. Other whole-field methods that have been developed include interferometry, moiré, holography, "speckle," and brittle coatings (also known as "brittle lacquers"). It seems important, therefore, that considerations be made about the relative merit of these methods for the solution of specific problems. Scientists however seem to have been even less concerned with the comparative evaluation of experimental methods than with the comparative evaluation of theoretical methods. A first attempt at this study was made by the author elsewhere[5] and further considerations were made more recently in another publication.[6]

The considerations to be made in this paper are to a certain extent the result of personal experiences. The reader can expect different opinions among research people. The relative values of methods will change with time as some develop more than others.

Comments will be made first on some theoretical methods, and then the attention will be concentrated on experimental approaches.

1. SOME MATHEMATICAL APPROACHES

The most direct, and oldest, theoretical methods consist in finding explicit closed form functions satisfying all established requirements of the stress and strain problem. Frequently, however, to obtain the solution some methods satisfy the differential equations identically, but satisfy only approximately the boundary conditions. Some other methods satisfy the boundary conditions identically but the differential equations only approximately. Finally, in some methods the differential equations and the boundary conditions are only approximately satisfied. The solution can be obtained using integrals to be numerically evaluated. Only in few instances solutions can be obtained in closed form.

Variational methods are energy methods of approximation which have proven to be fruitful in the solution of many two-dimensional structural problems, but are time consuming when applied to the solution of three-dimensional problems. These methods are averaging processes and smooth the steep gradients. They may be found objectionable when the determination of steep stress gradients is desired.

Four theoretical approaches are frequently considered: 1) Collocation (also called "boundary collocation" or point "matching"), 2) Conformal mapping, 3) Finite differences, and 4) Finite elements.

The first two methods are, in general, combination of analytical and numerical procedures while the last two are essentially numerical. The four approaches have been used in many fields of technology from vibrations and heat transfer to microwave situations and stress analysis studies. The

rate of convergence of the collocation method can be improved when a least squares criterium is added to the techniques.[7]

The conformal mapping technique has been used first in problems governed by Laplace's equation. The given, complicated shape is transformed onto a simpler geometrical configuration. Since Laplace's equation remains invariant under the transformation, the mathematical solution when the mapping function is available is straightforward in the transformed domain. For bidimensional elastic problems governed by Airy's biharmonic equation, Mushkelishvili[8] has developed the method to obtain the exact solution if the mapping function is known (usually in polynomial form).

The determination of the mapping function is in general the main difficulty of the conformal mapping approach. In some few cases, the function is known; in other cases it can be approximated in a fairly simple fashion, but in general one must make use of an extensive numerical procedure to obtain it. In case of a simply connected domain, finding the mapping function reduces to the solution of an integral equation[9][10][11] and for doubly connected domains (the case of a rocket motor cross-section) a system of coupled integral equations must be solved[12]. The mapping function must be very accurate in the neighborhood of points where stress concentration occurs. When this method is used it is frequently difficult to reproduce the geometry of sharp corners. This is one of the significant limitations of the method. Fortunately it is easier to approximate the internal corners than the external corners, and the internal corners are the most important in stress analysis.

Finite-difference methods reduce continuous systems to equivalent lumped-parameter systems. Instead of obtaining a continuous solution of the differential equation, the differential equation is replaced by a finite

difference equation and the approximate values of the solution are found at isolated points. The procedure leads to a system of n simultaneous linear algebraic equations when n discrete points are selected. A common way of solving this system of simultaneous equations is to use iteration or relaxation methods. When large stress gradients are present in the field, numerical instability and errors may take place. These methods have become practical since the advent of digital computers.

The method that has taken the greatest advantage of the new development in digital computers is the finite-element method. When this method is used the continuous structure is replaced by a large number of small elements and equilibrium and compatibility are applied to the whole of each of them. The method has found applications in many fields of mechanics and can be used when bodies have complicated boundaries and inhomogeneous, anisotropic, non-elastic properties. Smaller size elements can be used where gradients are high and storage capacity can be made very large, but practically there is a limit to the number of elements in the matrix to be used. Some investigators[4] propose hybrid techniques in which the local concentration is separated from the regular field, permitting an appreciable reduction in the number of elements. This technique has been introduced in some standard programs[13].

Depending on the type of problem to be solved finite-difference methods may be more or less efficient than finite-element methods. Considerations on the subject have been made by Bushnell[14] who calls attention to some of the finite-difference energy methods advantages. Key and Krieg[15] also compare both approaches and point out situations in which the finite-difference method may give better results. The introduction of arbitrary meshes in the finite-difference methods by Perrone[16] makes the method in many cases more

competitive with finite-elements methods. General consideration on finite-element methods from a historic perspective and also in comparison with finite-difference methods have been presented by Zienkiewicz[17] and by Oden[18].

2. GENERAL CHARACTERISTICS OF EXPERIMENTAL METHODS

There are three ways of approaching the solution of stress analysis problems: 1) measurements on the physical structure, or prototype; 2) measurements on a physical model of the structure; 3) computations on a mathematical model. Both physical and mathematical modeling imply an idealization of the structure, and the mathematical model requires knowledge of the boundary conditions. If functions are used, these conditions should be expressed analytically.

Some general characteristics of experimental methods will be recalled, to better understand what follows.

2.1 Model or Prototype

One of the first decisions to be taken is whether the analysis will be conducted on a model or on a prototype. Tests on models have all the great advantages of laboratory work in respect to work in the field, but require a sufficiently accurate similitude. Models do not always require the complete knowledge of the boundary conditions and can sometimes be reproduced directly from the prototype. Three-dimensional photoelasticity and three-dimensional moiré are essentially methods to be used on models. Electrical and mechanical strain gages can be used on models but more frequently are used on prototypes. Tests on prototypes can be conducted with electrical strain gages for instance, even if the loads applied to the structure are not

well-known. Two-dimensional photoelasticity requires the manufacturing of a model but in its form of photoelastic coatings it can be used on prototypes. Moiré and holography can be used both on prototypes and models, but at present they are used more frequently on models. "Speckle" methods are still being developed but it already seems that an important application may be to prototypes.

The investigator should not believe that because he takes measurements from a prototype he is not making any idealization. Any measurement of photoelastic coatings, moiré gratings, electrical resistance wires or foils, etc., involves an idealization. All it can be said is that this approach requires a lower degree of abstraction.

When the prototype to be measured is one of several, it should be representative of the group. Variations from item to item can be expected. Studies on welded joints are examples of this situation.

2.2 Point-by-Point or Whole-Field

Another of the important considerations is the one related to the size of the field of observation. Electrical, mechanical, and optical extensometers and strain gages give point-by-point and unidirectional information. Photoelasticity (for maximum shears), moiré, holography, interferometry and brittle coatings are whole-field methods. "Speckle" can be evaluated either way. Photoelasticity (for principal stress orientation) and grids can be considered as semi-whole-field methods.

2.3 Displacements, Strains or Stresses

The third important consideration is the information desired. Photoelasticity readily gives maximum shears, moiré gives components of displacements, brittle coatings give the direction and values of principal

stresses, holography gives displacements in particular in the direction of observation, strain gages give displacements between two points defining a base taken as reference, speckle photography gives the displacement vector.

It is difficult when photoelasticity is used to separate principal stresses. It is difficult when moiré is used to differentiate the displacements. It is difficult also to obtain displacements in the plane perpendicular to the direction of observation when holography is used. Photoelastic coatings are often not sufficiently sensitive and the separation of stresses can only be done with great difficulty. The direction of principal stresses is obtained precisely when brittle coatings are used but the precision of the value of the stresses is low.

Stress by definition takes place at a geometric point and is obtained as a limiting process. It cannot be measured, but it may come out of a theoretical analysis. There are many ways of defining strain. In general, strains are considered functions of derivatives of displacements. They cannot be measured either, but they can also come out of analysis. Displacements are motions. They are measurable.

3. EXPERIMENTAL METHODS FOR THE STRESS AND STRAIN ANALYSIS OF TWO-DIMENSIONAL ELASTIC PROBLEMS

Mechanical and electrical strain gages, photoelasticity, moiré, grids, brittle coatings and speckle will be considered.

The classic problem in two-dimensional elasticity has been for many years the determination of stress distributions in plates, with irregular geometries, and subjected to in-plane loading. Photoelasticity, in the form of whole-field isochromatics, has been used most frequently to solve this problem and gives directly in-plane normal stresses at the boundaries and maximum shears

throughout the field. Results obtained are catalogued in handbooks. Photoelasticity is still an excellent method to solve many problems of this kind, but frequently, depending on circumstances, the finite-element method may be preferred as being faster and less expensive. This is particularly true when separation of stresses is required, since methods to separate stresses supplementing photoelastic information are time consuming and in general yield less precise results than the information obtained from isochromatics.

If only the stress concentration factor is desired (the field of stress lacking interest) the problem becomes punctual and photoelasticity gives the answer readily. If the location at which the stress concentration occurs is known and a prototype is available, strain gages are frequently the most practical. In most laboratories electrical resistance strain gages are the ones used and they are certainly the best selection when strains may change with time. For static cases along straight lines of sufficient base length however, mechanical strain gages of the Huggenberger type may give information faster and be more reliable.

When the problem to be solved requires the determination of displacements, the use of moiré should be considered. In general, moiré is less sensitive than photoelasticity and to obtain strains the data has to be differentiated (computer programs are available), but displacements are determined readily.

Free surfaces of three-dimensional bodies can be considered in a first approximation as two-dimensional problems. When prototypes are available, either photoelastic coatings or brittle coatings can be used on them. Photoelastic coatings have two important limitations: low sensitivity and difficult separation of principal stresses, but they can be used several times under different loading conditions. Brittle coatings yield directly and with good precision, information on the direction of the principal stresses

but less precise information on their values. The coating records maximum tensile stresses which are frequently the most important in practical applications. Here also strain-gages may be practical if their number is not excessive.

When strains are relatively large (of the order of 2% or more) as in the case of plastics and elastomers, moiré and grids may be most practical, with grids to be used when strains are of the order of 15% or more. Considerations made in the next section may apply to this case also.

Holography in general does not seem to be well-suited for the determination of in-plane strains. It has however, with "speckle," the advantage of being applied directly to prototypes without requiring the printing of gratings on their surfaces (which is the case of moiré). The potentiality of "speckle" is still being developed, but recent improvements in the technique indicate that the method may have application to the determination of strains in prototypes.

4. EXPERIMENTAL METHODS FOR THE STRESS AND STRAIN ANALYSIS IN PLASTICITY PROBLEMS

Mechanical and electrical strain gages, and five whole-field methods will be considered: brittle coatings, photoplasticity, moiré, grids and photoelastic coatings.

4.1 Gages

Electrical strain gages of the resistance type can be used to solve problems in plasticity provided that: 1) the temperature is not too high (usually less than 700°F); 2) the time of observation is not too long (less than a few days); 3) the strain is not too large (less than about 0.02).

Mechanical strain gages are not subjected to these limitations, but their base length is much longer.

The important limitations of the method are that it gives only unidirectional and point-by-point information, and that they average the response over a relatively long base-length.

4.2 Brittle Coatings

Brittle coatings made with a resin as base can only be used at temperatures close to room temperature and when strains are relatively small (less than about 0.01). They permit the study of certain aspects of Lueder's lines and readily give the directions of the principal strains at a particular level of load.

4.3 Photoplasticity

Birefringence is used to study the behavior of materials, usually metals, modeling them with a plastic, usually celluloid or polycarbonate. It should be necessary to reproduce not only the stress-strain relationship but the flow conditions also. Some authors claim to have succeeded in this endeavor[19]. Actually, most of the time only the ideal plastic behavior is simulated. Flow conditions in biaxial states of deformation are difficult to reproduce and are frequently not well-known in the case of engineering materials. Plastic behavior after repetition of the load, or after the sign of the load has been changed, is not well-known for many engineering materials and even less so for the materials used to model them. To be practical the method requires that the relationship between birefringence and plastic strains, or stresses, be simple. The method would be useful if plastic deformations could be frozen and three-dimensional problems be solved.

4.4 Moiré

The most important advantage of moiré for plasticity studies is that it does not require the use of a model. Moiré has also the advantage of permitting applications at high temperature, although the technique for temperatures higher than 500°C is not yet well-controlled. The method gives displacements on the surface of the body to be analyzed, and the strains by differentiation, without requiring the use of any intermediate physical property. In general, it is accepted that the application of the method is limited by its low sensitivity (although much progress has been made recently on this point) but this limitation is insignificant in plasticity since strains are usually sufficiently large. It seems that efforts for future development should concentrate on this method.

4.5 Grids

When strains are very large (in general larger than 0.15) and the large number of fringes makes moiré evaluation difficult, the grid method is in general the most practical. The method is a semi-whole-field method since it gives a whole-field record, but the evaluation in general has to be done point-by-point. A quantitative comparison of these methods can be found in [20].

4.6 Photoelastic Coatings

Photoelastic coatings have, as moiré, some important advantages for plastic strain analysis. They are applied directly to the material to be analyzed and therefore do not require a model. They are sufficiently sensitive for plastic strains. An important limitation is that near discontinuities or at regions of high gradients, the error may be appreciable. To separate strains is always a difficult endeavor that lacks precision.

Moiré is better adapted in this sense. On the other hand, the coating is easier to apply to curved surfaces. After a critical analysis, some research people seem to prefer it[21].

4.7 Dynamic Photoplasticity

Comments made above about the relative value of photoplasticity and moiré apply also to problems in which the loading is dynamic. A discussion on this subject can be found elsewhere[22].

4.8 Three-Dimensional Applications

Several extrusion problems have been studied using the embedded grid method. The original bar is cut along the desired plane, and on the surface of this plane a grid network is engraved. The two parts are put back together and deformed. Measurements are taken on the grid before and after the deformation. The method works well on planes of symmetry.

If transparent materials are found with plastic behavior analogous to the one of the metal of interest, embedded grids could be used without limitation as to orientation.

5. EXPERIMENTAL METHODS FOR THE STRESS AND STRAIN ANALYSIS IN COMPOSITE MATERIALS

5.1 Photo-ortho-elasticity

A great effort has been spent recently to develop transparent orthotropic materials which exhibit birefringence when subjected to loads[23][24]. Although the strain-stress-birefringence relationship is complicated, stresses can be analyzed. Obviously, a different material should be manufactured for each different type of anisotropy. This method is attractive in the sense that it permits the integration of anisotropic studies in the classic body of photoelastic methods. The method corresponds to the same philosophy that

led to the modeling of the plastic behavior of metals, using polymers.

5.2 Photoelastic Coatings, Moiré, and Holography

There are three-whole-field alternatives to the photo-ortho-elasticity method: photoelastic coatings applied to the surface of the anisotropic bodies, and the methods of moiré and holography. The three of them seem to be more efficient than the method mentioned previously. The limitation of photoelastic coatings and moiré is in general lack of sensitivity, but in the case of composite materials most of the time deformations are appreciable and insure sufficient response for a precise analysis. The great advantage of the three methods is that they do not require the use of intermediate properties to obtain the necessary measurements. Photoelastic coatings give the field of shears easily, but they are affected by errors near the boundaries as a consequence of the difference in the Poisson's ratio of the coating and the composite material. Moiré gives the complete field of components of displacements which it is necessary to differentiate when strains are desired. Holography does not require the printing of a grating on the surface to be studied but it does require high stability in the test set-up and darkness during the recording. In addition, the procedure for the measurement of the displacements in the plane is usually not satisfactory and the determination of strains is also difficult.

6. METHODS IN DYNAMIC PHOTOELASTICITY

It is difficult to evaluate the methods available to solve problems in dynamics using photoelasticity. First, because there are many methods to choose from and then because the velocity of propagation of the deformations may vary within a large range of values.

6.1 Whole-Field Methods

The use of cameras, like the Fastax of Hycam, capable of taking up to about 30,000 pictures per second, is to be recommended when the velocities of the waves to be photographed are of the order of 50 m/s (soft materials like rubber). They are inexpensive and easy to use. When the time of exposure is of the order of 15 μs, a fringe moves only about 0.7 mm. For wave velocities of the order of 2000 m/s (epoxies) and provided the phenomenon is reproducible, the most suitable method is the use of still cameras receiving single flashes at controlled intervals during repeated experimental runs. The exposure time is about two-thirds of a μs and during this time a fringe moves about 1.5 mm. A practical alternative, although more expensive, is the camera of the Cranz-Schardin type which records 16 photographs with as many flashes each of a duration of about two-thirds of a μs. In these two cases, the film is stationary and may be about 10cm by 12cm in size which, in general, permits the recording of more precise photographs than the ones obtained on the moving narrow 8 or 16mm films. Another possibility is the use of ultrafast cameras, for instance the Beckman-Whitney, which record a few images obtained up to speeds of about one million pictures per second. In general, the price is higher, the technique necessary to use them difficult, and the quality of the image not as good as the one obtained with static images, or with the Fastax camera. Some investigators prefer the Beckman-Whitney camera when the object to be photographed is opaque and light is reflected by it. [(25)]

The determination of the individual principal stresses in dynamic problems requires a considerable effort which has seldom been attempted. Isoclinics can be recorded with repeated flashes as for the static case. The polariscope can also record oblique observations. However, the combination of photoelasticity with other methods like moiré or electrical strain gages seems more practical.

6.2 Point-by-Point Methods

All the methods mentioned above represent the dynamic phenomenon continuous in space but discontinuous in time (the interval between pictures). Other methods exist which represent the phenomenon as continuous in time but point-by-point, that is, discontinuous in space. The comparison between the two approaches can be illustrated considering the resolution of the images obtained with the individual flash (or the repeated flashes) which is of the order of 0.2mm, and the size of the spot observed in the point-by-point method which is about one order of magnitude larger. At present the smallest interval that can be obtained between static images with a flash is about 1 μs. Point-by-point methods in general are limited to the recording of birefringencies smaller than the first order, and this limits their sensitivity and also their precision. Methods using "streak" cameras can be related to this group of point-by-point methods.

Four whole-field methods have been mentioned above and among them the method of repeated flashes seems to be the most flexible, the easiest to use, and frequently the more advantageous for the price. All these methods, however, have to be used on a model and yield only maximum shears. Moiré has been applied very little to prototypes but its use would permit a more direct and more complete analysis of the phenomenon if enough response can be obtained. Little experience has been obtained up to now on the use of pulse lasers as light sources, but if found practical they will permit shorter exposure times and faster speed of recording.

7. METHODS IN THREE-DIMENSIONAL PHOTOELASTICITY

The choice has to be made usually between: a) "freezing" at elevated

temperature, b) scattered light, c) sandwich, d) "locking-in" at room temperature, 4) viscoelastic "locking-in."

7.1 "Freezing" at Elevated Temperature

"Freezing" at elevated temperature (about 120°C) is by far the most commonly used method. It is relatively easy to use and almost universal in its application. It has some limitations:

a) To obtain sufficient response to make the analysis practical, strains are frequently of the order of 0.02. Some research people consider this sometimes excessive when solutions in infinitesimal elasticity are desired.

b) Poisson's ratio of the materials used is close to 0.5 and therefore the solution obtained in principle is good only for incompressible materials and it is not possible to complete the information on maximum shears with information obtained from deformations.[26]

c) Frequently, an extraneous unwanted state of thermal stress appears at the loaded boundary as a consequence of the difference in the coefficient of thermal expansion of the model material (usually epoxies), and the material used to manufacture the loading devices (usually steel or aluminum). The presence of this extraneous state of stress has almost never been taken into consideration, although it can introduce appreciable errors. If for the manufacturing of the loading devices, plastics would be used exhibiting the same thermal coefficient of expansion of the epoxy of the model, but with a higher "freezing" temperature, that limitation could be overcome. In a recent work[27] it is claimed that such a plastic is now available.

d) The method is not applicable to dynamic problems nor to most types of thermal problems.

7.2 Scattered light

The scattered light method with several variations, attracts a great academic interest. It is not subjected to any of the limitations mentioned above for the "freezing" method. But scattered light finds two important difficulties:

a) The useful information comes as distance between fringes rather than as fringe orders.

b) Frequently, the light beam cannot reach the points of interest because of the presence of loading devices. This difficulty is basic and seldom mentioned by those who recommend the method. The difficulty is so great that many tests using scattered light are conducted on models previously frozen to permit the model to rotate freely in a polariscope without the encumbrance of the loading devices. This seems to remove from the method most of its advantages. On the other hand, the method presents the possibility of dynamic applications which are impossible when the freezing method is used. The light intensity, however, for this application should be very high. The method is also well-adapted for problems related to the determination of residual stresses in transparent materials. It is not necessary then to load the model which can be freely rotated in a tank with transparent walls and full of a liquid with the same index of refraction of the material of the body to be studied.

7.3 Sandwich

The sandwich method is limited to the study of a single plane in the

model but is not subjected to the objections related to the large strains to be applied to obtain sufficient response, nor to the objection of thermal effects produced by the loading devices. The method can also be used for dynamic studies.

7.4 <u>"Locking-in" at Room Temperature and Viscoelastic "Locking-in"</u>

The response obtained when "locking-in" at room temperature ("curing" method) or when viscoelastic temporary "locking-in" ("creeping" method) is used is sometimes larger than the response obtained using "freezing" at elevated temperature. An oven is then not necessary and no errors are introduced because of the possible presence of thermal stresses. To control the properties of materials to be used is, however, more difficult.

8. METHODS IN FRACTURE DYNAMICS

Fracture dynamics presents some specific problems. The most commonly used methods of analysis are dynamic photoelasticity and finite-elements.

Dynamic photoelasticity has the advantage of presenting readily whole-field information at several selected times. It has several disadvantages however for fracture dynamics applications. First, fracture is a non-reproducible phenomenon and this rules out the use of the repeated single flashes methods, and limits the usefulness of Cranz-Shardin cameras.

Then as was the case of photoplasticity and photo-ortho-elasticity a serious disadvantage of dynamic photoelasticity in its application to fracture dynamics is that the results obtained are generally speaking applicable only to the particular photoelastic model material used. Dynamic fracture resistance is a material property which depends on crack velocity. A correlation between the fracture dynamic model results obtained from

dynamic photoelasticity and the actual crack propagation results in the prototype, unlike its static counterpart, would be difficult to obtain due to the many material and geometrical variables involved.

While the onset of fracture and the crack extension rates are governed by the local stress intensity factor, dynamic crack propagation is influenced also by the global transient states. Global strain energy and kinetic energy distributions cannot be readily extracted from dynamic photoelasticity results while they are available in dynamic finite-element analysis.

It seems that in the near future finite-element codes, with proper singular elements which move with the crack tip, should match or surpass dynamic photoelasticity in its determination of dynamic stress intensity factors.

Despite the above odds, two-dimensional dynamic photoelasticity will continue to be useful in verifying dynamic finite-element codes under development for fracture dynamic applications. Such a role has been assumed by three-dimensional photoelasticity which is being used to spot check the many numerical computer codes available in solving three-dimensional problems in linear fracture mechanics. Dynamic finite-element codes which pass such screening can then be used in analyzing fracture dynamics of the actual hardware using actual material properties.

9. METHODS FOR THE ANALYSIS OF BENT PLATES

The experimental study of the bending of plates can be conducted using methods that yield directly either deflections or slopes. Curvatures are usually obtained from the slopes.

9.1 Deflection Methods

Among the deflection measurement methods, the simplest seems to be the projection grating method whereby one projects a grating onto the surface of the plate. The superposition of the deformed and undeformed grating images yields moiré fringes of equal deflection if the projection is a parallel one. The method of shadow-moiré is also simple. The grating has to be located near the plate surface and the moiré fringes are produced by the interferences of the grating and its shadow. Under certain conditions the fringes are contours of normal deflection. Holographic interferometry requires more delicate and expensive equipment. The projection moiré is suitable for relatively large deflections (0.1 mm and up), shadow-moiré for intermediate deflections (0.1mm to cm) and holographic interferometry for small deflections (smaller than 0.1 mm).

9.2 Slope Methods

Among the slope measurement methods the most widely used is Ligtenberg's whereby a mirrored model surface is used to view through a camera a diffusely illuminated grating. Images recorded before and after deformation give moiré fringes of constant slope. The zero state of the model plate is automatically preserved when the method is used. As originally presented the method could be applied only to small models statically loaded. Later developments have removed both restrictions.[28][29]

The Salet-Ikeda method projects a target of geometric pattern to a bent mirror-surfaced plate and receives its image through a pinhole at the focal point of an optical system. Other authors developed a family of "reverse" methods [30][31][32][33] whereby a target is placed at the focal plane of an optical system to code the light rays from the mirrored surface of a bent plate. The resulting fringes are contours of constant slope.

For all the above slope measurement methods the plate surface has to be mirrored. Recent developments[34][35] of the laser speckle method have removed this restriction. The approach adopted by Chiang and Jung[35] has shown that there is an analogy between the speckle method and the Ligtenberg method.

9.3 Curvature Methods

To obtain curvature contours most of the techniques shear two patterns. Duncan and Sabin use the pattern from Salet-Ikeda method, Saito, Yamaguchi and Hachimine use holographic shearing and Heise uses the deformed grating image of Ligtenberg set-up. An optical spatial filtering technique was developed by Chiang and Bailangadi[36] where a deformed grating obtained by a Ligtenberg arrangement is spatially filtered at different locations to yield curvature contours in succession. Chiang and Kao, and Assa obtained curvature contours in a one-step process[30][33]. All these curvature measurement methods require that the model surface be specularly reflective.

9.4 In-Plane Displacements

A review of several methods used for the direct determination of strains in plates can be found in[37]. A unique approach is offered by Durelli and Parks[38] whereby the in-plane surface displacement of the plate is obtained directly. In this technique a grating is printed on the surface of the plate and deformed. A film master grating adheres to the surface by a film of oil.

All things being equal, the curvature contouring method should be chosen over the slope measuring method which, in turn, should be chosen over the deflection measuring method because of the inaccuracy associated with the differentiation of experimental data. All these methods, but

Durelli and Parks[38] are based on the theory of bending of plates. If plates are thick, or have discontinuities, it may not be legitimate to determine strains from deflections or slopes and in-plane displacement methods should be used.

10. PHOTOELASTIC METHODS AND MOIRÉ

Moiré as well as photoelasticity can be used either on models or on prototypes. Photoelasticity applied to prototypes takes the form of a coating covering the surface to be studied. Photoelasticity yields directly maximum shears, moiré yields components of displacement.

a) Two-dimensional static photoelasticity in models can be very sensitive and very precise. The method is self-sufficient for the determination of the magnitude and directions of maximum shears but requires supplementary information to separate the stresses. Moiré sensitivity and precision, when applied to models, are not in general as high as those of photoelasticity.

b) The use of moiré on prototypes for the solution of two-dimensional static, elastic problems, despite the techniques of fringe multiplication available today is limited in general in its application by the 40 ℓpmm maximum density of the available gratings. Without multiplication the maximum response of moiré is about one fringe per inch when applied to a structural steel subjected to elastic strains. A differentiation of the information is necessary to obtain the strains but the strain tensor at every point is completely determined. Multiplication and differentiation techniques are becoming more practical.

c) Photoelastic coatings can be applied to prototype surfaces of complicated shape. In general, they lack sufficient sensitivity, their

maximum response being about one fringe when applied to structural steel subjected to elastic strains. Separation of stresses is complicated and lacks precision.

d) Up to now, moiré in general cannot be applied to curved surfaces except when they are developable, or the radius of curvature is very large.

e) Photoelasticity applied to the solution of bent plate problems gives membrane stresses but the determination of bending stresses is much less practical.

f) Reflective moiré[38] permits the easy determination of strains in bent plates. The problem can be completely solved as long as the final curvature of the plate is not too large.

g) For elastic finite strains as those taking place in transparent rubbers, photoelasticity and moiré easily yield useful information. Beyond strains of the order of 0.15, grids are more practical because of the large number of photoelastic and moiré fringes to be analyzed.

An important difference to be kept in mind between the two methods is that moiré yields directly components of displacements without the need of any intermediate physical property. And displacements are the basic concept on which are based all the developments of the theories of mechanics of continua. The images of deformed gratings are geometric images. Strains are derived from them directly.

On the other hand, photoelasticity yields directly the changes in indices of refraction, most frequently the changes in the differences between indices of refraction. For the common cases of elastic static small deformations at room temperature the calibration is easy. However, if the calibration has to be conducted for relatively high temperature, or

the loads are applied rapidly, the deformations are large, or irreversible, then the calibration can be very complicated and the transformation of birefringence measurements into stresses or strains becomes less precise and subjected to several errors.

11. THE DESIGN OF POLARISCOPES

A large number of different kinds of polariscopes have been developed and built in the last 30 years. Many ideas have been used to design them. Some of these ideas have important consequences in the use of the instruments. Comments on these points follow.

11.1 With or Without Lenses

Some polariscopes are built with optical systems made of many lenses. The designer made a great effort to obtain a light that is parallel when going through the model, and to correct optical errors. At the other extreme it is possible to find polariscopes with only two lenses and, for those using diffused light, with only one lens the one necessary to record a photograph. The cost of lenses usually increases rapidly fast with their size, and their quality is less satisfactory. In general, polariscopes with many lenses have optical fields appreciably smaller than the fields of diffused light polariscopes. This means that with diffused light polariscopes bigger models can be observed and in general the precision of the measurements can be improved. Some polariscopes have lenses inside the field of polarized light. In this case, any residual birefringence in the lenses is added to any observation of the model.

The position of points on the model can be determined by displacing the loaded model and measuring the displacement with a micrometer, or can

be determined by displacing the ground glass receiving the image. Both methods are inconvenient. It is possible to draw a grid on the model with an appropriate Indian ink (for instance, Rotring) the width of which is about 0.2mm. It is also possible to inscribe a grid using a milling machine. The localization of points of the grid on a photograph can be obtained with a precision of about 0.1mm.

The relative precision of lens polariscopes and diffused light polariscopes has been studied by Leven[39]. In general, it is better for diffused light polariscopes. These polariscopes are much less expensive and easier to manufacture and to use than the lens polariscopes. Lens polariscopes however permit easier enlargement of the image and its projection on a screen.

One of the great advantages of diffused light polariscopes is the possibility of moving its components about large models loaded in complicated fashions. This requires that the components on the two sides of the model be separated. Some polariscopes permit the enlargement of the image on the screen, others permit the photographic recording of the image. Diffused light polariscopes require a long focal length lens and the image is usually photographed. Enlargements are obtained from the photographic film.

11.2 Whole-Field or Point-by-Point

Classic polariscopes permit the recording of isoclinics and isochromatics in the whole field. They appear as dark fringes of light extinction. Other polariscopes receive point-by-point information and use measurements of light intensity and phase difference. In these polariscopes usually the analyzer is made to rotate continuously. The recording can be automatic and is in general much more complicated than the recording in classic polariscopes. Methods used with these polariscopes measure fractions of orders of

interference and only with difficulty the orders of the fringes can be determined. As a consequence of this fact, they are recommended in general for measurements on glass or on plexiglas. In classic polariscopes, the maximum order of fringes recorded is usually about 20 and the measurement of the birefringence can be obtained with a precision of about 0.01 of fringe order (or 0.005 with special precautions). In rotating analyzer point-by-point polariscopes, the precision of the measurements is about the same but the maximum number of fringes should not be more than 0.5. The precision in the measurement of isoclinics is 1 degree and 2 degrees, respectively.

12 RELATIVE EFFICIENCY OF EXPERIMENTAL METHODS AND FINITE ELEMENT METHODS

In most European and American applied mechanics laboratories when the determination of a stress distribution is desired, the question arises of the relative efficiency of experimental methods on one hand and the so-called finite-element methods on the other hand. There is no unique answer to this question because the answer depends on:

1) The nature of the problem to be solved.
2) The availability of the computers or the instruments necessary, respectively, for the finite-element methods and for the experimental methods.
3) The degree of suitability of the computers on one hand, and the machines and instruments on the other hand, to solve the practical problems.
4) The experience of the operator.

Surveys have been conducted in numerous American and European laboratories on the present state of the art. It seems that:

1) Finite-element methods, with a few exceptions, have almost completely replaced photoelasticity in the solution of two-dimensional elastostatic problems.

2) Finite-element methods are also well-suited for the solution of rotationally symmetric three-dimensional problems.

3) Photoelasticity is, in general, the fastest and least expensive method to solve three-dimensional problems when the geometry and the loads of the bodies to be analyzed do not exhibit rotational symmetry.

4) When the prototypes are three-dimensional and made of materials with low Poisson's ratio, as is the case in many civil engineering structures, the matter is debatable since up to now models for experimental determinations are manufactured with materials which have a high Poisson's ratio. For problems in which the stress distribution is strongly influenced by small variations in Poisson's ratio like some second boundary problems in elastomers with very high Poisson's ratio, the precise simulation with a physical model may be also difficult to obtain.

5) Experimental methods are in general better suited for the solution of wave propagation problems. If the material has time-dependent properties, however, finite-element methods may be better suited.

6) Moiré and grids are in general the most practical methods to use for the solution of two-dimensional plasticity problems.

7) Moiré and holography are well-suited to solve problems dealing with vibrations of plates, which one among these methods is the

most efficient depends on the particular case.

8) Holography is well-suited for deflection measurements on shells. Finite-elements are well suited in general for determinations of strains in shells.

9) Photoelasticity permits the most efficient optimization of the geometry of discontinuities in plates loaded in the plane.

It is also important to point out that finite-element methods require the knowledge of the boundary conditions. On the other hand, experimental methods can solve problems on prototypes (for instance a plane in flight) even if the boundary conditions are not well-defined. Frequently, if only deformations are desired, it is not necessary either to know the constitutive law of the materials. If models are manufactured, the boundary conditions do not have to be known with the same degree of precision that is required when theoretical methods are used.

Sometimes in the United States, and more frequently in Europe, results obtained with finite-element methods are available but they do not seem to be completely trusted and an experimental verification is desired. In Europe, it is frequently stated that photoelasticians have more work now than before the development of finite-element methods, as a consequence of this need of verification.

There are few laboratories in the world in which experimental methods and finite-element methods are mastered simultaneously. It would be very useful if in these laboratories an attempt be made to weigh quantitatively the comparisons mentioned above. If more convenient, well-defined problems could be solved independently in laboratories of approximately the same scientific level but specializing either on experimental methods or on finite-element methods. The comparison should include considerations on the

time necessary to solve each problem, on the cost, the degree of confidence, and the obtained precision. Among the available comparisons of experimental and finite-element solutions is a work by Laura et al[40] in which stress concentrations were evaluated for two cases: first, the finite-element results were compared with results obtained using photoelasticity for stress concentration factors at the boundary of a slot with semicircular ends in a plate of finite width subjected to uniaxial stress. The experimental results were about 10% larger than the finite-element results with the difference increasing as the length of the slot increased. Secondly, a comparison was made between the finite-element results and an exact solution obtained by Muskhelishvili for stress concentrations at the edge of a corrugated hole in an infinite plate subjected to equal biaxial loading. The exact values are about 7% higher than the finite-element values. In both cases elements of different sizes were used to increase the accuracy near the fillets. Obviously, smaller size elements will give more accurate results.

Another evaluation was conducted by Becker and Brisbane[41] on two classic problems, the point loading the edge of a half-space (Boussinesq problem) and the stresses around an elliptic hole. The finite-elements used were rather crude and presently available programs would permit a higher precision. The authors found that the finite-element results were consistently higher than the exact values due to an apparent increase in stiffness of the finite-element representation of the continuum. It is also obvious from their analysis that errors may become larger where the gradients are large.

It may be pointed out that it is common in both approaches, photoelasticity and finite-elements to extrapolate results obtained near the boundary, to the boundary itself. This operation is dangerous in both cases,

but the possible error in photoelasticity is bound by the order of the next not visible fringe. Values obtained from finite-element analysis, or from point-by-point strain gages may be affected by a larger error.

13 MATHEMATICAL AND EXPERIMENTAL IDEALIZATIONS

Mathematical approaches, whether "exact" or approximate, use a mathematical model that idealizes the geometry of the body, the loading conditions applied to it, and the physical behavior of the matter of which it is made. The object is to take advantage of mathematical relationships which should correlate with the desired physical behavior.

Many experimental methods, like photoelasticity, use models also. They work frequently like computing machines. The results obtained from the idealized model are to be applied to the physical prototype. In both cases there is a dangerous passage from the idea to the reality.

Gages, or coatings, applied to the prototypes are also in a way idealizing the physical situation. They all give values of the desired quantities, averaged over a base length, or over a surface, or over a volume. The refined experimental stress analysis methods usually stop at what happens in elements of sizes smaller than 0.001 in[42]. Since stress analysis is conducted most of the time for engineering purposes, the underlining assumption is that what occurs inside elements of that size is not relevant to failure.

It should also be understood that mathematical and physical idealizations are possible when stress and strain distributions are desired, but the parallel problem of the determination of physical properties cannot be modeled. Gages of different kinds unavoidably have to be used to determine constitutive equations and in no way can their measurement be replaced with mathematical approaches. Polariscopes have to be used when the birefringent

properties of transparent material is desired. Residual stresses depend strongly on the mechanical and thermal properties of the particular material and can seldom be modeled. The detection of cracks and flaws also is determined in each particular material, and has to be done experimentally.

ACKNOWLEDGMENTS

The preparation of this paper has been supported financially by the Office of Naval Research (contract N00014-76-C-0487), and the John F. Dodge chair in engineering at Oakland University. The author is grateful to the monitor N. Perrone for his understanding and encouragement.

The manuscript has been read by several scientists in the field of stress analysis, among whom A. Assa, F. P. Chiang, J. der Hovanesian, A. Kobayashi, P. Laura, C. Parr, V. J. Parks and R. J. Sanford have made important suggestions. The author likes to acknowledge these contributions.

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It is necessary to weigh the advantages and limitations of the numerous methods available today for the analyses of displacements, strain, and stresses. Systematic studies are lacking on this subject. In this paper after comments are made on several theoretical approaches, in particular approximates methods, attention is concentrated on experimental methods and considerations are made on the relative merits of: 1) photoelasticity, moiré, brittle and photoelastic coatings, gages, grids, holography and

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speckle to solve two-dimensional problems; 2) photoplasticity and moiré to solve problems in plasticity; 3) photo-ortho-elasticity, photoelastic coatings and moiré to solve anisotropic problems; 4) the methods used to solve problems in dynamic photoelasticity; 5) the methods used in three-dimensional photoelasticity, and 6) projected gratings, shadow-moiré and holography to analyze bent plates. General considerations are also made on the relative merit of whole-field methods, in comparison with point-by-point methods. Finally, an attempt is made at evaluating merit of the finite-element method in respect to the experimental stress analysis methods.

Advantages of particular methods and instruments are emphasized in the paper: whole-field methods, diffused light polariscopes, moiré to solve problems in plasticity and in anisotropy, repeated flashes to solve reproducible problems in dynamic photoelasticity. It is also shown that in general the "freezing" method is the most practical and the most precise to solve three-dimensional problems. The range of usefulness of projected gratings, shadow moiré and holography to analyze plates is indicated. It is pointed out that the finite element method has replaced, or will soon replace, two-dimensional static photoelasticity and will probably also do so in fracture dynamics applications. On the other hand, photoelasticity competes frequently with advantage with the finite-element method, in the solution of wave propagation problems, optimization problems, and three-dimensional problems. The potential of the method of "speckle" is pointed out.